# SEED: An Effective Model for Highly-Skewed Streamflow Time Series Data Forecasting 

Yanhong Li, Jack Xu, David C. Anastasiu

Santa Clara University

## Problem: predicting long-term streamflow values with rain off data

$$
\left[x_{1}, x_{2}, \ldots, x_{T}\right] \in \mathbb{R}^{T} \rightarrow\left[x_{T+1}, \ldots, x_{T+H}\right] \in \mathbb{R}^{H}
$$

$x_{1}$ to $x_{T}$ : the input sequence
$x_{T+1}$ to $x_{T+H}$ : the output sequence
In our research: $H=3$ * 24 * $4=288$, with majority of normal values and much fewer extreme values which cause the data skewness to one side.

## Challenges:

- Long-range dependencies.
- Rare but important extreme values; very imbalanced data.

Goal:

- An end-to-end extreme-adaptive model;
- Long sequence forecasting (predicted length = 288);

Dataset:

- Four groups of hydrologic datasets from Santa Clara County, CA.
- Namely Ross, Saratoga, UpperPen, and SFC, named after their respective locations.


## Dataset with high skewness and kurtosis scores:



High skewness and kurtosis scores indicate that there is significant deviation from a normal distribution in our data!

Four streams: Ross, Saratoga, UpperPen, and SFC. Hydro year: from September to May.

| Statistic / Stream | Ross | Saratoga | UpperPen | SFC |
| :---: | ---: | ---: | ---: | ---: |
| mean | 2.91 | 5.77 | 6.66 | 20.25 |
| max | 1440.00 | 2210.00 | 830.00 | 7200.00 |
| min | 0.00 | 0.00 | 0.00 | 0.00 |
| median | 0.17 | 1.00 | 3.20 | 1.20 |
| variance | 597.22 | 711.09 | 452.90 | 12108.14 |
| skewness | 19.84 | 19.50 | 13.42 | 18.05 |
| kurtosis | 523.16 | 697.78 | 262.18 | 555.18 |

# Motivation: achieving the best overall prediction performance, without sacrificing either the quality of normal or of extreme predictions. 

Root Mean Square Error (RMSE)

Mean Absolute Percentage Error (MAPE)

## Proposed Methods:

Framework: Segment-Expandable Encoder-Decoder (SEED) model, which is the first to integrate segment representation learning with a multi-tiered encoder-decoder framework.

Importance-enhanced sampling strategy: embedded within the SEED model, allowing it to skillfully identify key features and trends in datasets.

Representation Learning: A unique regularization strategy that incorporates a Kullback-Leibler divergence regularization loss term across multiple stacked layers, thereby increasing the model's robustness against anomalous events with divergent distributions.

## Background: Piecewise Linear Representation (PLR)


> PLR splits a series into several segments such that the maximum error of each segment does not exceed a threshold;
> Prior work: PLR describes the linear relationship of the multi-segment representation, mainly works as a preprocessing step to reduce both the space and computational cost of storing and transmitting time series.
> Our work: inspired by PLR, SEED learns nonlinear segment representations for heavily skewed long term time series.

## SEED framework :


$>$ Comprises three core components: embedding, encoder, and decoder.
$>$ The encoder generates a unique hidden state and a cell state which serve as the initial values for the corresponding layers in the decoder.
$>$ Each decoder layer is assigned a distinct task, as they represent the mean value distribution of different lengths of subsegments in the predicted sequence.

## Convolutional Embedding Layers:



CNN Layers:

- different kernel sizes to extract features at different spatial scales.
- subsequent tanh activation function;
- lower level: larger kernel sizes, capturing broader patterns and global context.
- higher level: smaller kernel sizes, capturing local patterns and fine grained details.


## Decoder Architecture:



- First level: the output length is 4 , which is meant to predict the mean values of 4 segments, each of which contains 288/4 $=72$ points in the forecasted series.
- Second level: the 4 outputs are expanded to16, each represents the mean value of 288/16 = 18 points.
- Expansion: $\langle\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}\rangle$ becomes $\langle\mathrm{a}, \mathrm{a}, \mathrm{a}, \mathrm{a}, \mathrm{b}, \mathrm{b}, \mathrm{b}, \mathrm{b}, \mathrm{c}, \mathrm{c}, \mathrm{c}, \mathrm{c}, \mathrm{d}, \mathrm{d}, \mathrm{d}, \mathrm{d}\rangle$, in the high-dimensional hidden space.
- By predicting the mean value of different length sub-segments, extreme values are represented and spread across multiple levels in the hierarchy, leading to higher mean values in the segments containing them.


## Multiple-Objective Loss Functions:

$$
\begin{gathered}
\operatorname{KL}(p \| q)=\sum p(x) \log \left(\frac{p(x)}{q(x)}\right), \\
\mathcal{L}_{i}=\operatorname{KL}\left(\operatorname{softmax}\left(p_{-} m_{i}\right), \operatorname{softmax}\left(g_{-} m_{i}\right)\right), \\
\mathcal{L}=\operatorname{RMSE}(\hat{y}, y)+\lambda \times\left(\sum_{i=1}^{k} \mathcal{L}_{i}\right),
\end{gathered}
$$

## Motivations:

> Kullback-Leibler divergence loss acts as a regularization term that encourages the model to match the target distribution while balancing the sequence generation loss.
$>p_{-} m_{i}$ represents the predicted segment mean values in the ith layer, while $g_{-} m_{i}$ is the vector of computed ground truth mean values for the segments in the ith layer. .

## Importance-Enhanced Oversampling Policy:

Input : Dataset with training and inference sequences
Output: Oversampled training set
Procedure Oversampling;
while training set size is not satisfied do
Randomly sample a sequence including training and inference sections;
if maximum value in inference section $>T$ then
Mark sequence as important;
Move maximum value to the middle of the inference section of the sequence;
foreach index $I$ in the sequence with step size $S$ do Sample starting at $I$;
Add sampled sequence to oversampled training set;
end
end
else
Add sequence to oversampled training set; end
end

## Steps:

> important sequences: maximum values in the inference section of the series exceed a threshold T;
$>$ moving the maximum value to the middle of the inference section;
$>$ multiple iterations of sampling from the beginning with a specified step size $S$.

## Baselines:

- FEDFormer
- InFormer
- NLinear
- Dlinear
- NEC+
- EnDecoder, the common encoder-decoder model built with LSTM layers.

Main results:

| Methods <br> FEDformer | Metric <br> RMSE <br> MAPE | Ross |  |  | Saratoga |  |  | Upperpen |  |  | SFC |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{array}{r} \text { All } \\ 6.49 \\ 2.49 \end{array}$ | $\begin{array}{r} \text { High } \\ 30.82 \\ 5.27 \end{array}$ | Low <br> 2.51 <br> 2.04 | $\begin{array}{r} \text { All } \\ 6.85 \\ 2.26 \end{array}$ | $\begin{array}{r} \text { High } \\ 11.95 \\ 1.50 \end{array}$ | Low <br> 4.59 <br> 2.60 | $\begin{array}{r} \text { All } \\ 2.38 \\ 1.02 \end{array}$ | $\begin{array}{r} \text { High } \\ 17.68 \\ 2.37 \end{array}$ | $\begin{gathered} \text { Low } \\ 1.07 \\ 0.90 \end{gathered}$ | $\begin{array}{r} \text { All } \\ 24.15 \\ 2.81 \end{array}$ | $\begin{array}{r} \text { High } \\ 94.28 \\ 1.70 \end{array}$ | Low <br> 6.68 <br> 3.09 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Informer | RMSE | 9.14 | 31.00 | 5.56 | 4.89 | 13.64 | 1.01 | 5.33 | 16.26 | 4.40 | 19.00 | 85.40 | 2.46 |
|  | MAPE | $5.45$ | 5.80 | 5.39 | 0.73 | 1.40 | 0.43 | 4.21 | 2.70 | 4.34 | 0.54 | 0.71 | 0.49 |
| Nlinear | RMSE | 5.84 | 32.12 | 1.54 | 4.98 | 14.61 | 0.70 | 1.74 | 15.07 | 0.61 | 18.43 | 83.31 | 2.26 |
|  | MAPE | $1.62$ | 4.89 | 1.09 | 0.75 | 1.74 | 0.31 | 0.57 | 1.69 | 0.47 | 0.87 | 1.03 | 0.83 |
| Dlinear | RMSE | 6.90 | 30.96 | 2.97 | 4.06 | 7.63 | 2.48 | 3.25 | 14.01 | 2.33 | 23.64 | 79.76 | 9.65 |
|  | MAPE | $2.79$ | $4.03$ | 2.58 | 1.31 | 0.85 | 1.51 | 2.04 | 1.68 | 2.07 | 4.02 | 1.04 | 4.76 |
| NEC+ | RMSE | 9.33 | 38.34 | 4.58 | 1.95 | 5.55 | 0.35 | 1.94 | 13.92 | 0.92 | $\underline{16.39}$ | 76.63 | $\underline{1.38}$ |
|  | MAPE | $4.53$ | 8.33 | 3.91 | 0.21 | 0.30 | 0.17 | 0.80 | 0.84 | 0.80 | 0.55 | 0.61 | 0.54 |
| NBeats | RMSE | $\underline{5.16}$ | 30.09 | $\underline{1.08}$ | 3.60 | 9.44 | 1.01 | 1.23 | $\underline{13.20}$ | $\underline{0.21}$ | 31.47 | 95.33 | 15.55 |
|  | MAPE | $\underline{1.25}$ | 3.17 | 0.94 | $0.70$ | 1.21 | 0.47 | 0.25 | 0.78 | 0.20 | 3.24 | 0.88 | 3.83 |
| EnDecoder | RMSE | 5.58 | 30.81 | 1.45 | $\underline{1.93}$ | 5.69 | $\underline{0.26}$ | 2.95 | 16.33 | 1.80 | 17.46 | 79.04 | 2.11 |
|  | MAPE | 1.62 | 3.72 | 1.28 | 0.16 | 0.29 | 0.11 | 1.81 | 2.34 | 1.76 | 0.84 | 0.96 | 0.80 |
| SEED | RMSE | 4.23 | 29.74 | 0.05 | 1.67 | 5.14 | 0.12 | 1.07 | 12.83 | 0.07 | 14.44 | 70.04 | 0.59 |
|  | MAPE | 0.11 | 0.53 | 0.04 | 0.09 | 0.19 | 0.05 | 0.10 | 0.57 | 0.06 | 0.20 | 0.42 | 0.14 |

- Univariate Long-Term $(\mathrm{h}=288)$ Series Forecasting Results.
- Over 1600 test points in the test set were inferenced on all datasets.
- The best results are in bold and the second best results are underlined.
- "All" represents the average RMSE of all test samples compared with the ground truth. "High" means larger than the mean value; "Low" includes test samples lower than the mean value.

In comparison to the three second-best models (NEC+, Nbeats and EnDecoder), SEED achieved, on average, relative RMSE reduction of $31.44 \%, 34.68 \%$, and $29.67 \%$ across the datasets.

Example comparisons with the second best baselines:







Effect of the Importance-Enhanced Oversampling Policy:

(a) RMSE of UpperPen

(b) RMSE of Saratoga

- To evaluate the impact of this policy, we increased the threshold T while simultaneously decreasing the step size S .
- Increasing the threshold T and decreasing the step size $S$ had a positive impact on the results.
- There is an optimal threshold T value beyond which the policy's effectiveness plateaus.

Effect of the KL Regularization Terms \& Segment Expanding:





We just use $T=4, \mathrm{~S}=8$ as an example:

- T4S8: 5-layer without regularization loss terms.
- T4S8-3L: 3-layer SEED with regularization loss terms.
- T4S8-Regu: 5-layer SEED with regularization loss terms, which gives the best result.

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Q \& A
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